A Survival Analysis of Wildfire Outcomes

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Background

- Rising suppression costs
  - Pent-up fuel supply from historically aggressive suppression policy
  - Growing wildland urban interface
  - Climate change
- Costs scrutinized by Congress and relevant agencies
- Increasing emphasis on cost efficiency

Problem Statement

- Incident Commanders allocate suppression resources in an uncertain dynamic environment
- Decisions based on information updated daily
- **Research Question**: How do stochastic factors, such as weather and values-at-risk, impact three cumulative wildfire outcomes: time, cost, and area?

Problem Statement

- Incident Commanders allocate suppression resources in an uncertain dynamic environment
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- **Research Question**: How do stochastic factors, such as weather and values-at-risk, impact three cumulative wildfire outcomes: time, cost, and area?

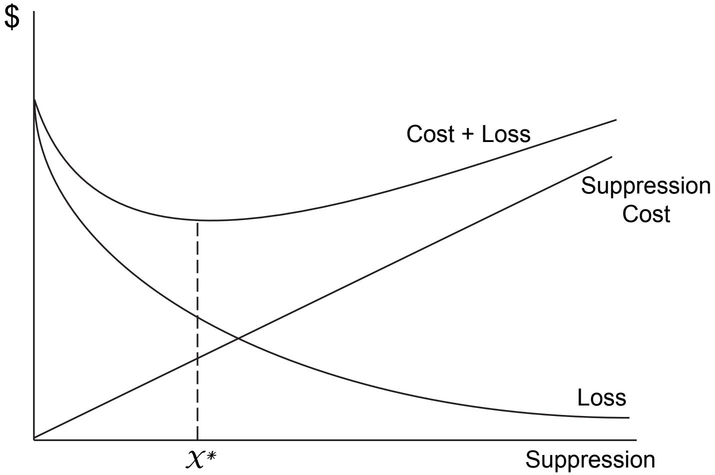
Goals and Objectives

- Develop theoretical motivation of incident commander's suppression resource allocation
- Estimate reduced form hazard model to identify the impact of stochastic factors on three wildfire outcomes:
  - **Cost** - Cumulative suppression costs accrued over the course of a fire
  - **Area** - Cumulative area burned over the course of a fire
  - **Time** - Total time a fire burns before termination
- Provide a framework for forecasting wildfire outcomes

Literature

- Intra-fire analysis
  - Parks (1974)
  - Donovan and Rideout (2003)
- Inter-fire analysis
  - Holmes et al. (2008)
  - Donovan and Brown (2005)

Framework



Incident Commander

$$\underset{r_t}{\text{Max}} \quad \sum_{t=0}^{T} \beta^t U_t \left[L(r_t; v_t, z_t), C(r_t; v_t, z_t) \right]$$

where:

- $r_t \in \mathbb{R}_+^N$ – suppression resources
- $v_t \in \mathbb{R}_+^N$ – values at risk (i.e., houses, timber, etc.)
- $z_t \in \mathbb{R}_+^K$ – external factors (i.e., weather, region, etc.)

Incident Commander Necessary Condition

$$U_L L_r + U_C C_r = 0 \quad \forall\ t$$

- where $U_L < 0$, $U_C < 0$, $L_r < 0$, and $C_r > 0$
- IC equates marginal utility of avoided losses with marginal disutility of incurring costs
- Cumulative outcomes are a function of stochastic factors and values at risk over time

$$C^*(\{v_t, z_t\}_{t=0}^{T}), \quad A^*(\{v_t, z_t\}_{t=0}^{T}), \quad T^*(\{v_t, z_t\}_{t=0}^{T})$$

Data

- Merged dataset including 11,566 fires between 2001 - 2008:
  - Incident Command Survey
  - National Interagency Fire Management Integrated Database

List of Variables

Weather & Env. Characteristics	Fire Characteristics	Lagged Outcomes*
Wind	Threatened Commercial	Destroyed Commercial
Temp	Threatened Residential	Destroyed Residential
Humidity	Threatened Other Bldgs.	Destroyed Other Bldgs.
Season	Cause	Evacuation
Elevation	Prescribed	Injuries
Slope	Escaped	Fatalities
Aspect	Growth Potential	
Latitude	Percent Contained	
Region	Suppression Strategy	

Why Hazard Model?

- Why not linear regression with dependent variable = time?
  - Censoring
- Why not probit or logit?
  - Length of duration is uncertain

Hazard Rate

$$\lambda(t) = \Pr(t < T < t + \Delta t | T \geq t) = \lambda_0(t) e^{x(t)'\beta}$$

- Natural to think of an accumulation of time
- Similar concept with cumulative cost and area

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Selected Results

$$100 \cdot [exp(\beta_i \cdot x_i) - 1] = \% \text{ change on } \lambda$$

	Cost	
	Est.	Pval
Wind	-3.88	0.00
Temp	-1.90	0.00
Humidity	-1.08	0.00
Elevation	-6.77	0.00
FS Region 2 (RM)	-4.91	0.81
FS Region 5 (PSW)	-67.11	0.00
Lightning	4.92	0.58
Potential Evacuation	-72.12	0.00
Threatened Residential	-0.05	0.24
Injuries_{t-1}	-12.91	0.06
% Contained	4.03	0.00
Contain (Strategy)	4.74	0.85
High Gr. Potential	-69.38	0.00
	Fires	Obs
	1,455	10,754

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	Cost		Area	
	Est.	Pval	Est.	Pval
Wind	-3.88	0.00	-2.72	0.00
Temp	-1.90	0.00	0.02	0.93
Humidity	-1.08	0.00	0.26	0.10
Elevation	-6.77	0.00	0.27	0.88
FS Region 2 (RM)	-4.91	0.81	44.15	0.05
FS Region 5 (PSW)	-67.11	0.00	50.95	0.01
Lightning	4.92	0.58	-21.20	0.00
Potential Evacuation	-72.12	0.00	-7.12	0.39
Threatened Residential	-0.05	0.24	-0.02	0.27
Injuries$_{t-1}$	-12.91	0.06	0.01	0.16
% Contained	4.03	0.00	1.34	0.00
Contain (Strategy)	4.74	0.85	65.10	0.03
High Gr. Potential	-69.38	0.00	-76.42	0.03
	Fires	Obs	Fires	Obs
	1,455	10,754	1,981	8,990

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	Cost		Area		Time	
	Est.	Pval	Est.	Pval	Est.	Pval
Wind	-3.88	0.00	-2.72	0.00	-0.50	0.36
Temp	-1.90	0.00	0.02	0.93	0.04	0.02
Humidity	-1.08	0.00	0.26	0.10	-0.09	0.45
Elevation	-6.77	0.00	0.27	0.88	-6.64	0.00
FS Region 2 (RM)	-4.91	0.81	44.15	0.05	89.25	0.00
FS Region 5 (PSW)	-67.11	0.00	50.95	0.01	79.85	0.00
Lightning	4.92	0.58	-21.20	0.00	-26.01	0.00
Potential Evacuation	-72.12	0.00	-7.12	0.39	-50.68	0.00
Threatened Residential	-0.05	0.24	-0.02	0.27	-0.14	0.05
Injuries$_{t-1}$	-12.91	0.06	0.01	0.16	-4.95	0.32
% Contained	4.03	0.00	1.34	0.00	2.50	0.00
Contain (Strategy)	4.74	0.85	65.10	0.03	-32.65	0.02
High Gr. Potential	-69.38	0.00	-76.42	0.03	-45.50	0.00
	Fires	Obs	Fires	Obs	Fires	Obs
	1,455	10,754	1,981	8,990	2,036	20,787

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Potential Evacuation	-72.12	0.00	-7.12	0.39	-50.68	0.00
Threatened Residential	-0.05	0.24	-0.02	0.27	-0.14	0.05
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Contain (Strategy)	4.74	0.85	65.10	0.03	-32.65	0.02
High Gr. Potential	-69.38	0.00	-76.42	0.03	-45.50	0.00
	Fires	Obs	Fires	Obs	Fires	Obs
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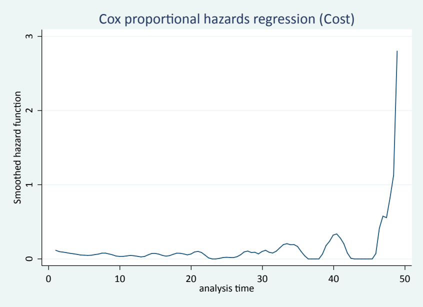
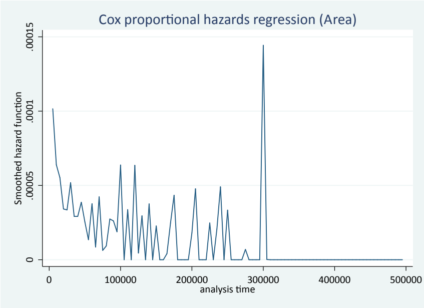
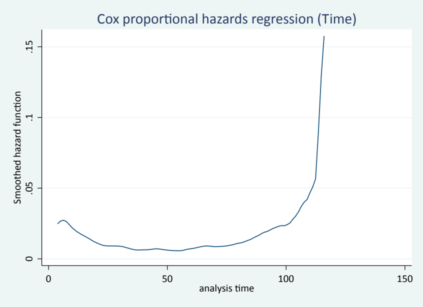
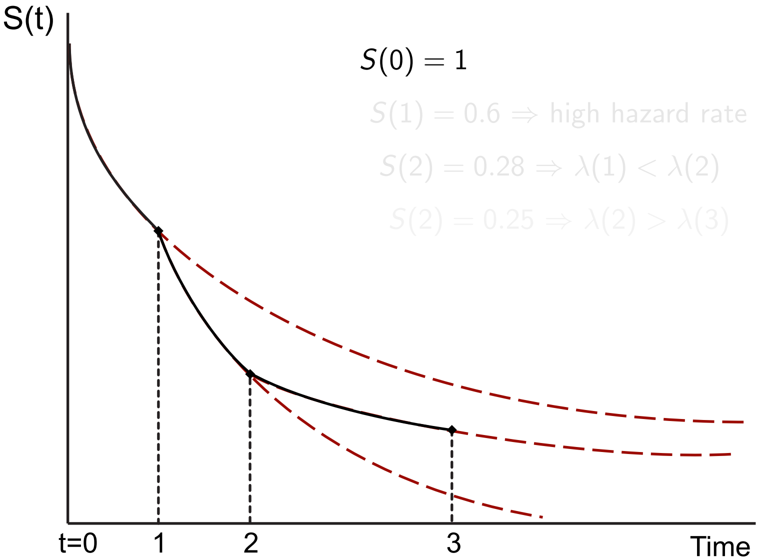
Cox Regression Hazard Estimate







Example Fire



$S(0) = 1$

$S(1) = 0.6 \Rightarrow$ high hazard rate

$S(2) = 0.28 \Rightarrow \lambda(1) < \lambda(2)$

$S(2) = 0.25 \Rightarrow \lambda(2) > \lambda(3)$

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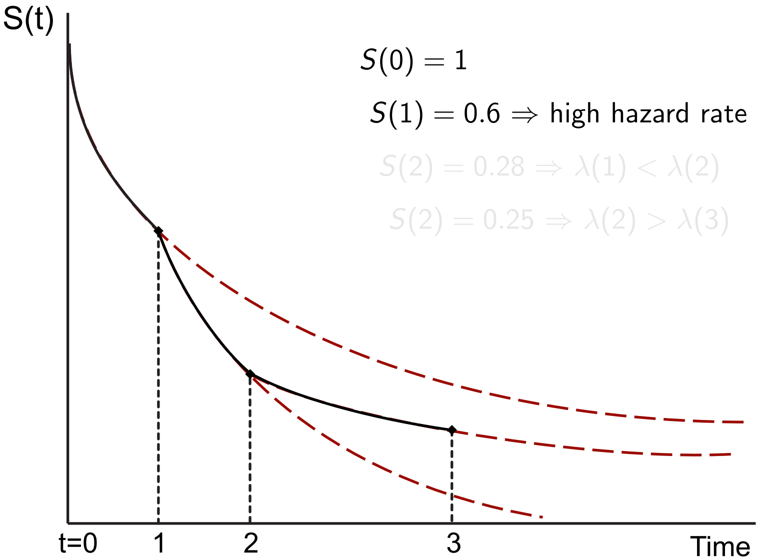

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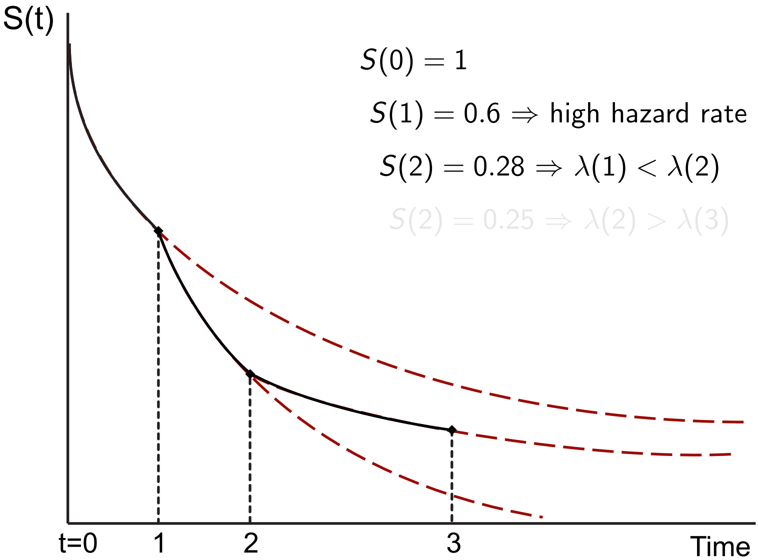

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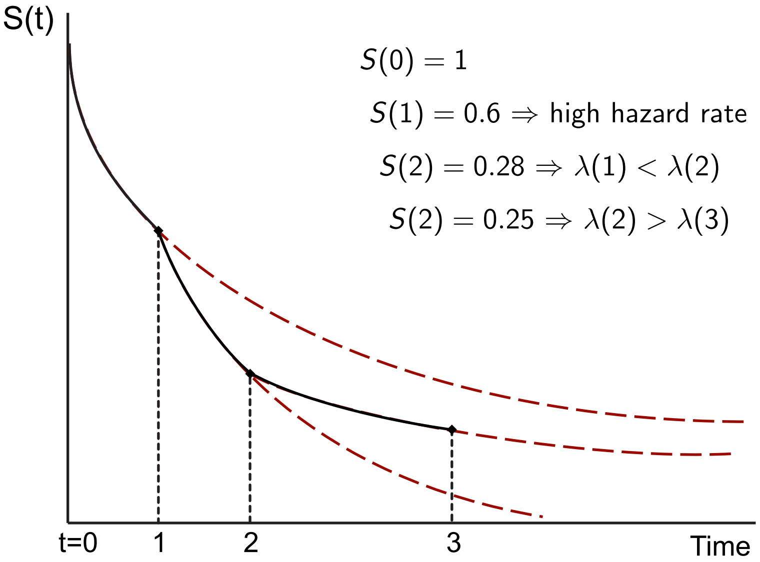


Implications

- Model results may be used to forecast probabilistic fire behavior
- Help IC make informed decisions by reducing uncertainty
- Strategic plan requests probability of success (WFSA)
- Need to include resource allocation decisions

Incident Commander i

$$\underset{r_{it}}{\text{Max}} \quad \sum_{t=0}^{T_i} \beta^t U_{it} \left[L(r_{it}; v_{it}, z_{it}), C(r_{it}; v_{it}, z_{it}) \right]$$

$$\text{s.t.} \quad R_{it} \geq r_{it} \geq 0 \; ; \; \forall \; t$$

where:

- $r_{it} \in \mathbb{R}_+^N$ – suppression resources
- $v_{it} \in \mathbb{R}_+^N$ – values at risk (i.e., houses, timber, etc.)
- $z_{it} \in \mathbb{R}_+^K$ – external factors (i.e., weather, region, etc.)
- $R_{it} \in \mathbb{R}_+^N$ – resources available from regional command center

Incident Commander Necessary Condition

$$\beta \left(U_L L_r + U_C C_r \right) = \lambda \quad \forall\, i, t$$

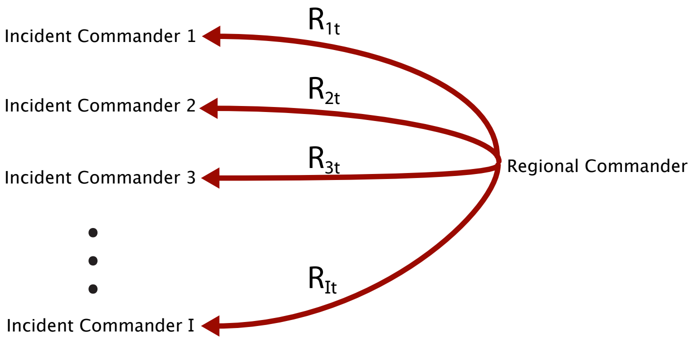
- where $U_L < 0$, $U_C < 0$, $L_r < 0$, and $C_r > 0$

$$r^*_{it}(R_{it}, v_{it}, z_{it}) \;,\; \lambda^*_{it}(R_{it}, v_{it}, z_{it})$$

- $\lambda_{it} \in \mathbb{R}^N_+$ represents the marginal utility of the set of resources
- λ signals the need for resources to regional command

Chain of Command

Regional commander delegates resources to individual fires



Regional Commander's Problem

$$\max_{R_{it}} \quad \sum_{t=0}^{T_A} \beta^t W_t \left[\lambda_{1t}(R_{1t}, v_{1t}, z_{1t}), \ldots, \lambda_{It}(R_{It}, v_{It}, z_{It})\right]$$

$$\text{s.t.} \quad B \geq \sum_{t=0}^{T_A} \sum_{i=1}^{I} P_{it}^{'} R_{it}$$

- Where $P_{it} \in \mathbb{R}^N_{++}$ are the prices/rental rates of all possible resources
- Resources are allocated among $i = 1, \ldots, I$ competing fires on a daily basis

Regional Commander Necessary Condition

$$\beta^t \frac{\partial W_t(\cdot)}{\partial \lambda_{it}} \frac{\partial \lambda_{it}(\cdot)}{\partial R_{it}} = \mu P_{it} \quad \forall\ t,\ i$$

- where $W_\lambda < 0$ and $\lambda_R < 0$
- Regional commander strives to allocate resources such that the shadow value is equal across fires and time
- Cumulative outcomes are a function of stochastic factors and values at risk over time

$$C_i^*(\{v_{it}, z_{it}\}_{t=0}^{T_i}), \quad A_i^*(\{v_{it}, z_{it}\}_{t=0}^{T_i}), \quad T_i^*(\{v_{it}, z_{it}\}_{t=0}^{T_i})$$

Estimation

Likelihood Function with Censoring:

$$L = \prod_{i=1}^{n} f_i(t_i|\mathbf{x}_i)^{\delta_i} S_i(t_i|\mathbf{x}_i)^{1-\delta_i} \quad \text{where } \delta \in \{0,1\}$$

$$L = \prod_{i=1}^{n} \lambda_i(t_i|\mathbf{x}_i)^{\delta_i} S_i(t_i|\mathbf{x}_i) \quad \text{note } \lambda(t) = \frac{f(t)}{S(t)}$$

Cox Partial Likelihood Method (Cox, 1974):

$$PL = \prod_{i=1}^{k} \frac{\exp(\mathbf{x}_i'\beta)}{\sum_{j\in R(t_i)} \exp(\mathbf{x}_j'\beta)}$$

where $k \leq n$ are the uncensored failure times

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