Valuation of Washington DNR Timber Harvest Contracts under Price Uncertainty

A Real Options Approach

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Outline

Washington DNR Timber Harvest Contracts

Brief Overview

- Types:
  - `Lump-Sum Sale`: All timber sold to highest bidder for cash.
  - `Scale Sale`: All timber sold to highest bidder by volume or weight.
  - `Contract-Harvest Sale`: Log sorts from a stand sold individually.
- Properties:
  - Fixed contract length
  - Harvest any time prior to conract expiry
  - Uncertainty about future timber prices
- Fees omitted for clarity.

American Call Options

Basic Properties

- Derivative contracts commonly traded in financial markets.
- Characteristic Properties:
  - `Length`: Fixed at time of sale.
  - `Strike Price`: Fixed at time of sale.
  - `Number of Shares`: Fixed at time of sale.
  - `Call Property`: The right, but not the obligation, to purchase an asset at strike price.
  - `American Property`: Early exercise any time prior to expiry date.

Timber Harvest Contract as Real Option

- Timber harvest contract is an American call option on the value of timber.

Property	**American Call**	**Harvest Contract**
Contract Length	Fixed	Fixed
Strike Price	Strike Price	Harvest Cost
Underlying Asset	Stock	Timber
Exercise Time	Anytime before T	Anytime before T

- The forestry firm has the right to purchase the timber at the harvest cost and sell it in the market at the prevailing price.
- Techniques for valuation of financial options can be adapted to valuation of timber harvest contracts.

American Option Valuation

- Early exercise possibility greatly complicates American option valuation.
- Closed-form solutions of problems pertaining to valuation American options are limited to special cases.
- Solutions approximated by the use of numerical methods.
- Valuation of American options is an area of active research.
- A popular technique is the LSM algorithm introduced by Longstaff and Schwartz in 2001.

Least-Squares Monte Carlo Algorithm I

- At each point in time, prior to harvesting, the forest owner must make a decision:

$$\max \left[CF_t;\ \mathbb{E}\left(\sum_{t+1}^{T} d_i\, CF_i \middle| P_t, P_{t-1}, \ldots \right) \right]. \qquad (1)$$

  i.e. Choose the larger of:
  - Profit from harvesting today, $CF_t = Q_t\,(P_t - C)$, or
  - Discounted expected profit from harvesting in the future, $\mathbb{E}\left(\sum d_i\, CF_i | \ldots\right)$.
- The conditional expectation term is critical to contract valuation yet can rarely be calculated exactly.

Least-Squares Monte Carlo Algorithm II

- The LSM algorithm provides an estimate of the $\mathbb{E}\left(\sum d_i\, CF_i \middle| \ldots\right)$ term in equation 1.
- The LSM starts at time T and moves backward in time:
  1. Generate price vectors P_t and P_{t+1}.
  2. Use P_{t+1} to calculate CF_{t+1} a vector of cashflows from harvesting at time $t+1$.
  3. Calculate $\hat{\mathbb{E}}\left(\cdot|\cdot\right)$ by regressing CF_{t+1} on a set of basis functions of P_t.
  4. Substitute $\hat{\mathbb{E}}\left(\cdot|\cdot\right)$ into equation 1 and decide whether to harvest at time t or not. Update record of harvest values accordingly.
  5. Repeat previous steps until reaching contract starting date.

Stochastic Price Model

- Many models for price behavior are available.
- Choice is driven by available historical data and economic theory.
- The model used for this example is a log mean-reverting stochastic process:

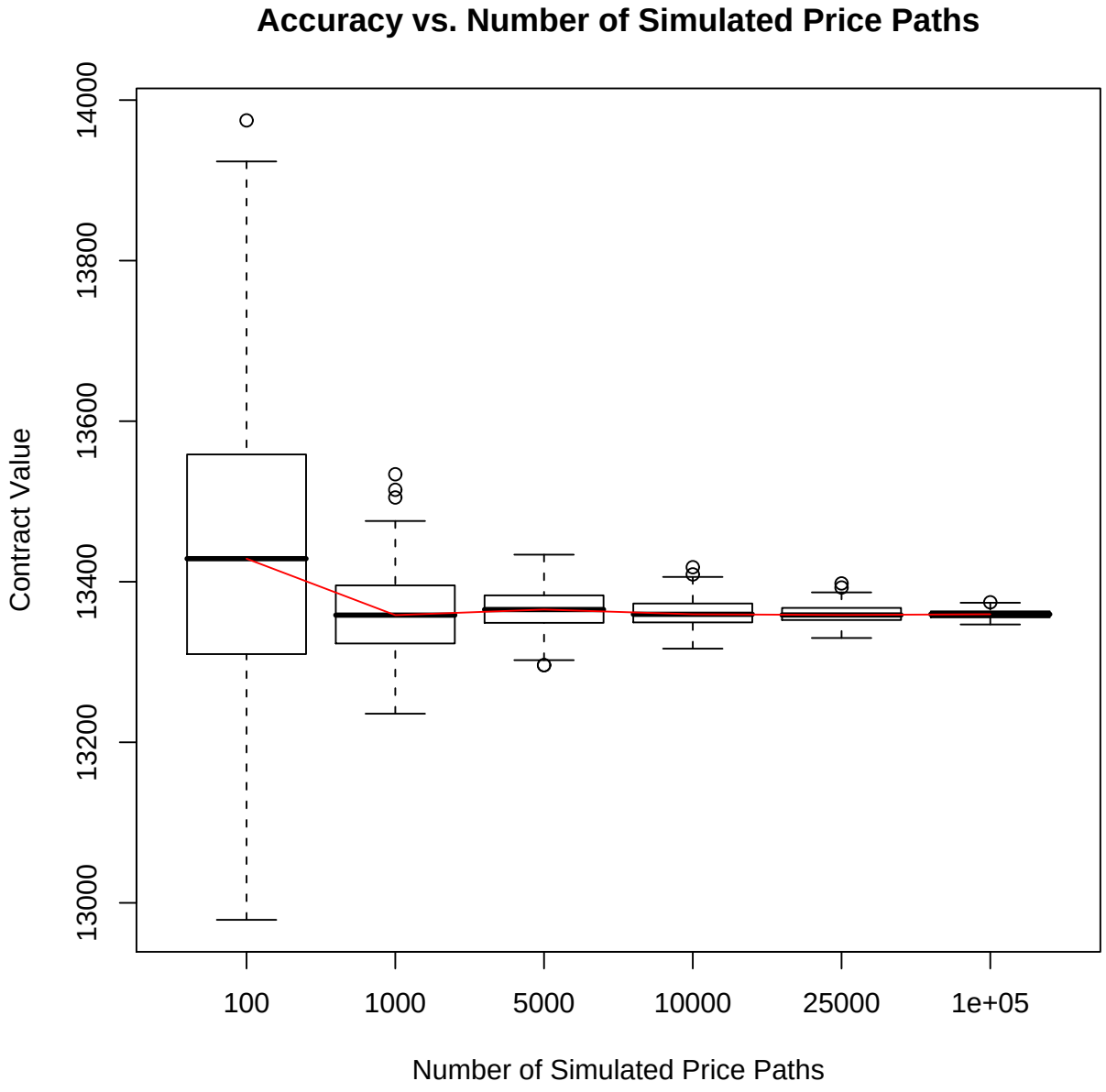
$$dS_t = \kappa(\mu - \ln S_t)\, S_t\, dt + \sigma\, S_t\, d\mathbb{W}_t, \tag{2}$$

where κ is the rate of mean reversion, μ is the log of long term price, σ represents price volatility and $d\mathbb{W}_t$ is an increment of the Wiener process.

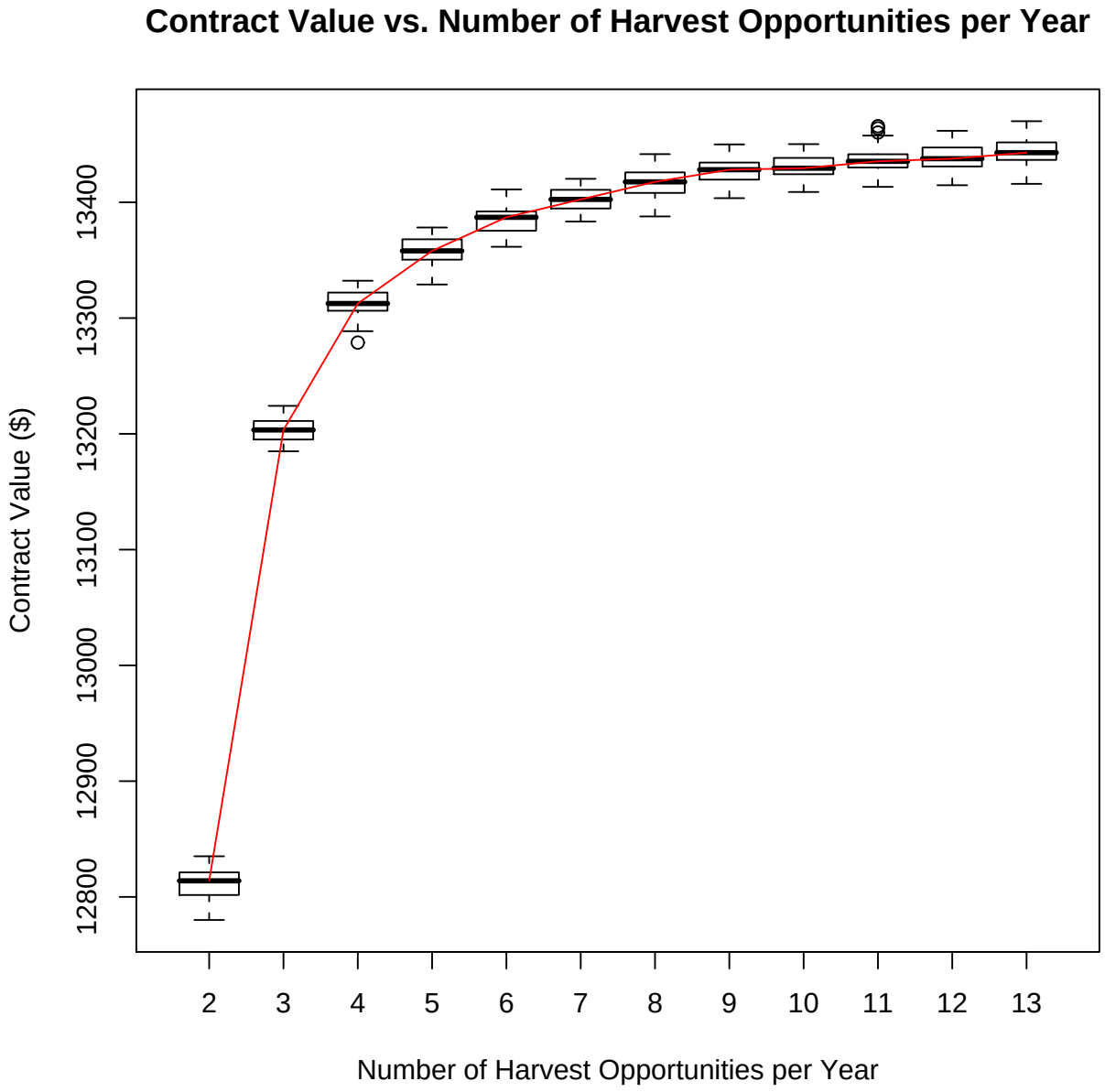
Parameter Settings

- `Harvest Cost` equal to $100 per MBF.
- `Rate of Discount` assumed to be 5% per year.
- `Yield Function` assumed high-yield site in Western Washington.
- Initial `Stand Age` equal to 50 years.
- `Contract Length` equal to 3 years.
- `SDE Parameters`: long-term trend $\mu = 6.11$, rate of reversion $\kappa = 0.40$, volatility $\sigma = 0.25$
- `Initial Price` equal to $450 per MBF.

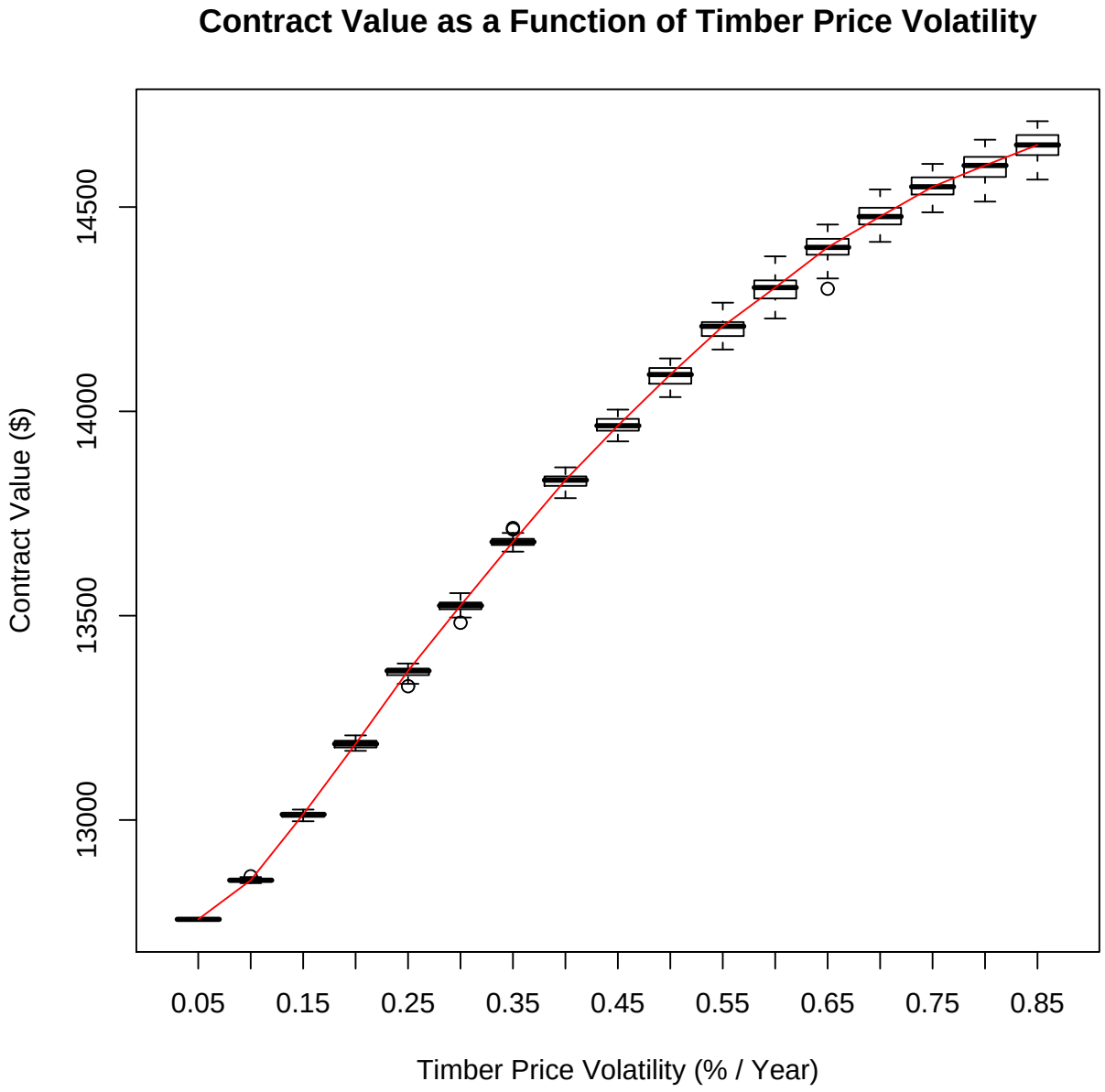
Solution Accuracy



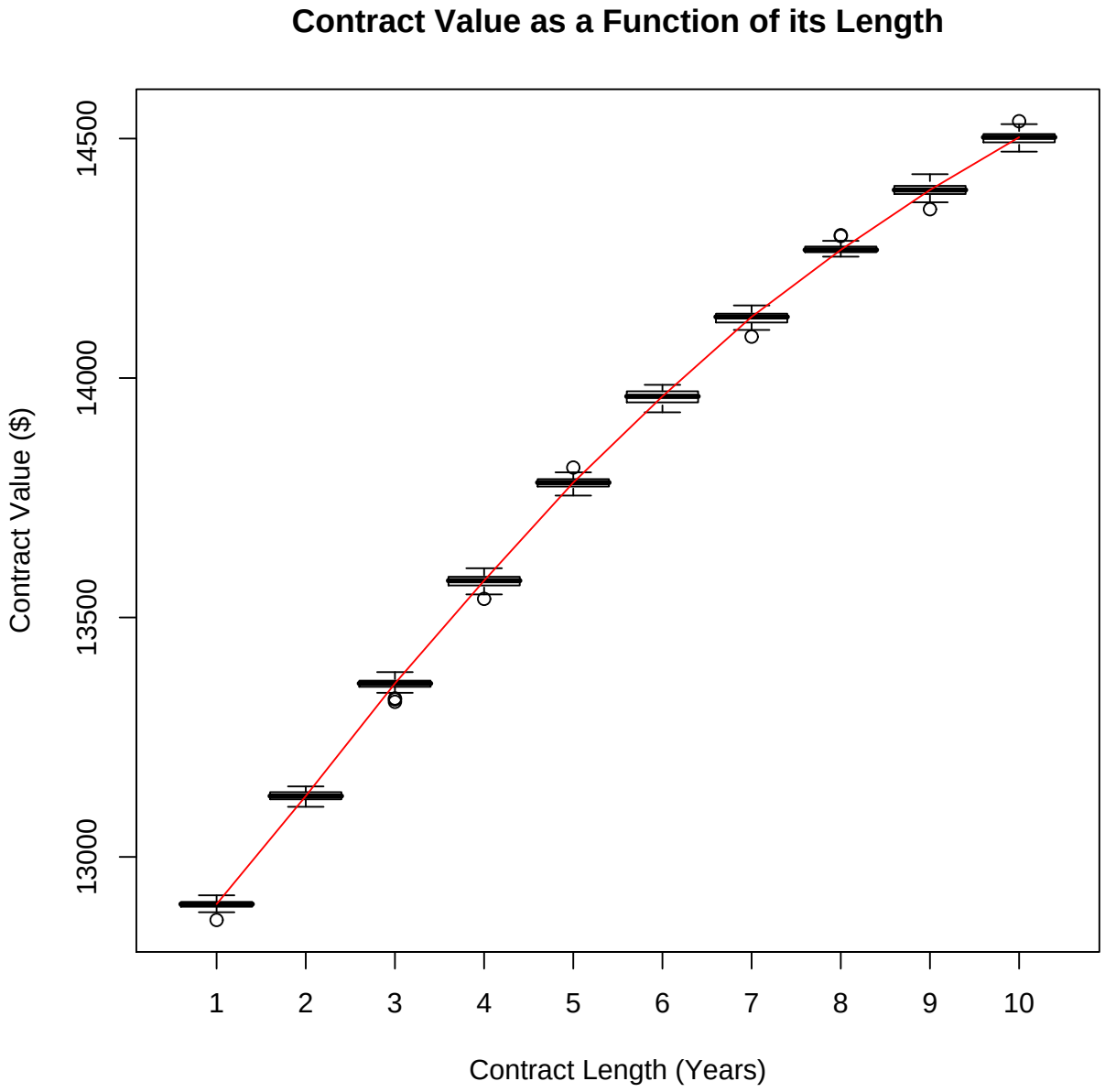
Number of Exercise Opportunities



Timber Price Volatility



Contract Length



Summary

- The Washington DNR timber harvest contract is, in essence, an American call option on the value of timber.
- An enhanced form of the LSM algorithm provides an effective tool for timber harvest contract valuation.

- Outlook
  - Replication of the results for a variety of stochastic price models.
  - Application to a large set of past contracts.

References I

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Publication Pending.