Forest Taxation under the Generalized Faustmann Formula

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- Since the seminal work of Fred Fairchild in 1935, the classic Faustmann formula has served as the basis of forest taxation research.
- In 1983, I published a comprehensive set of formulas for land expectation value under various forms of forest taxation, discussed the relationship between various form of forest taxes and analyzed their impact on the optimal rotation age.

- In 1998, to overcome the stringent assumptions that stumpage prices, stand volume, regeneration cost, annual incomes and expenses, as well as interest rate remain the same from rotation to rotation under the classic Faustmann formula, the generalized Faustmann formula for land expectation value was developed.

- The intriguing questions is how similar and /or different are formulas for forest taxation under the classic and generalized Faustmann formula as well as the impact of various forest property taxes on forest management decisions.

- In this presentation, I will present the case of the general unmodified property tax under the generalized Faustmann formula and show that other property taxes are special cases of the general formula.
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- Under the classic Faustmann formula, the land expectation value with the general unmodified property tax (UPT) is
- $LEV_{w/upt} = \left(\frac{r+y}{r+x}\right) \frac{[V(t) - Cexp((r+y)t)]}{[\exp((r+y)t) - 1]}$
- where r is the interest rate, x is the tax rate on the value of the land, y is the tax rate on the value of the trees, V(t) is the stumpage value, C is the regeneration cost.

- When y=x, as is commonly the case,
- $LEV_{w/upt} = \left(\frac{r+y}{r+x}\right) \frac{[V(t) - Cexp((r+y)t)]}{[\exp((r+y)t) - 1]}$
- becomes
- $LEV_{w/upt} = \frac{[V(t) - Cexp((r+x)t)]}{[\exp((r+x)t) - 1]}$

- When y=0, the general unmodified property tax
- $LEV_{w/upt} = \left(\frac{r+y}{r+x}\right) \frac{[V(t) - Cexp((r+y)t)]}{[\exp((r+y)t) - 1]}$
- Becomes that of the site value tax
- $LEV_{w/svt} = \left(\frac{r}{r+x}\right) \frac{[V(t) - Cexp(rt)]}{[\exp(rt) - 1]}$

- It is well know that under the classic Faustmann formula, the unmodified property tax acts as an increase in interest rate and will shorten the rotation age.
- On the other hand, a site value tax will not affect the optimal rotation age.

- Under the generalized Faustmann formula, the land expectation value with unmodified property tax is
- $$LEV_{1\,w/upt} = \left(\frac{r+y}{r+x}\right)\frac{[V(t) - Cexp((r+y)t) + (LEV_2 - LEV_{1\,with\,UPT})\,]}{[\exp((r+y)t) - 1]}$$
- Notice that the formula under the generalized Faustmann formula is almost identical to that of the classic version, except the difference between LEV_2 and LEV_1 w/upt at the end of the numerator.

- $$LEV_{1\,w/upt} = \left(\frac{r+y}{r+x}\right)\frac{[V(t) - Cexp((r+y)t) + (LEV_2 - LEV_{1\,with\,UPT})]}{[\exp((r+y)t) - 1]}$$

- When $LEV_2 = LEV_1$ w/upt the generalized Faustmann formula becomes the classic version.

- When there is no tax on the value of the standing timber the $LEV_{1\ w/upt}$ becomes that with a site value tax (SVT)
- $$LEV_{1\ w/upt} = \left(\frac{r+y}{r+x}\right) \frac{[V(t) - Cexp((r+y)t) + (LEV_2 - LEV_{1\ with\ UPT})]}{[\exp((r+y)t) - 1]}$$
- $$LEV_{1\ w/svt} = \left(\frac{r}{r+x}\right) \frac{[V(t) - Cexp(rt) + (LEV_2 - LEV_{1\ with\ svt})]}{[\exp(rt) - 1]}$$

- Further manipulations of the above formulas yield
- $LEV_{1\ with\ UPT} = \frac{r+y}{(r+x)[(\exp((r+y)t)-1]+(r+y)}[V(t) + \int_0^t A_j \exp(r+y)\,(t-j)dj - Cexp(r+y)t + LEV_2]$

- $LEV_{1\ with\ SVT} = \{\frac{r}{[(r+x)\ \exp(rt)-x]}\}[V(t) + \int_0^t A_j(r(t-j))dj - Cexp(rt) + LEV_2]$

Tax impact

- An unmodified property tax acts as an increase in interest rate and shortens the rotation age
- With forest taxation under the generalized Faustmann formula
- $$LEV_{1\,w/upt} = \left(\frac{r+y}{r+x}\right) \frac{[V(t) - Cexp((r+y)t) + (LEV_2 - LEV_{1\,with\,UPT})]}{[\exp((r+y)t) - 1]}$$
- It shortens the rotation age even more.

- For a site value property tax,
- $LEV_{1\ w/svt} = \left(\frac{r}{r+x}\right) \frac{[V(t) - Cexp(rt) + (LEV_2 - LEV_{1\ with\ svt})\ ]}{[\exp(rt) - 1]}$
- the tax will definitely affect the optimal rotation age and the magnitude depends on the difference between LEV_2 and $LEV_{1\ w/SVT}$.

t	P(t)	Q(t)	V(t)	LEV(t)		LEV1	LEV1	LEV1
					when LEV2=		$ 700.00	per acre
					Tax rate =		1%	
(year)	($/cu.ft.)	(cu.ft./ acre)	($/acre)	($/acre)		WITHOUT	WITH A	WITH A
0						TAX	SVT of 1%	UPT 0f 1%
11	0.300	1,821	546.30	278.08		496.15	459.18	429.05
12	0.301	2,134	642.33	320.81		505.38	465.56	431.50
13	0.302	2,430	733.86	347.87		509.29	467.12	429.16
14	0.303	2,725	825.68	366.81		510.65	466.47	424.60
15	0.304	3,036	922.94	382.93		511.84	465.77	419.93
16	0.305	3,335	1,017.18	391.29		509.50	461.98	412.28
17	0.306	3,637	1,112.92	396.17		505.73	457.03	403.53
18	0.307	3,919	1,203.13	394.57		498.29	448.89	391.83
19	0.308	4,210	1,296.68	392.11		490.58	440.63	380.08

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Conclusion

- Formulas for forest taxation under the generalized Faustmann formula looks very similar to that of classic Faustmann formula.
- However, the difference between LEV_2 and LEV_1 at the end of the numerator causes the tax to affect the optimal rotation differently.
- Thus, in additional fiscal neutrality, the tax burden of various forms of forest taxation needs to be carefully examined.